Robotics (Robótica)
First Trimester (Primer Trimestre) 2013-2014
Model Solution For Localization Exercises

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1 Seminar 1

1.1 Problem Description

A robot is in a tiled corridor (18 tiles), the corridor has red and green doors, the robot has no knowledge where it is when its starts, it could be in any of the tiled squares and has only a sensor that can discriminate if there is a door in front of it or not (but not the color of the door). The robot has the a map about the corridor as Fig. 1

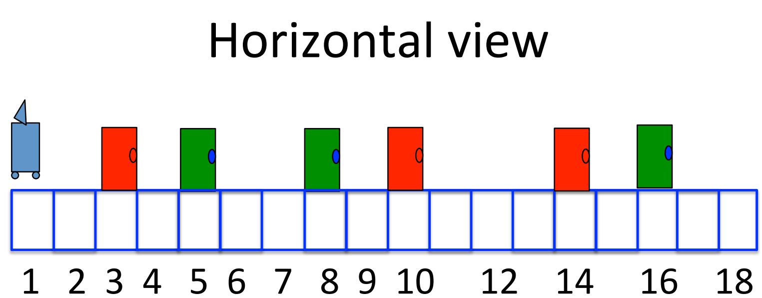

Figure 1: The map of the environment with red and greed doors that act as landmarks.

1. What is the state space of the robot?
2. The robot moves one tile to the right and sees a door. If we represent the belief of where the robot is as a probability distribution of the 18 tiles, what is the probability distribution now?
3. The robot moves now 2 more tiles to the right and sees another door after this move. What are the values of the probability distribution know?
4. Repeat the exercise with a robot that can also sense the color of the doors, if the first door found is red and the second is green.

1.2 Solution Method

1.2.1 What is the state space of the robot?

The robot can be in any of the 18 tiles or squares. Thus, the state space of the robot is the 18 positions $\{x = 1, x = 2, x = 3, x = 4, x = 5, x = 5, \ldots, x = 18\}$

1.2.2 What is the belief of the robot initially?

We would indicate the initial state as happening at time 0, i.e, $t = 0$. Because the problem in this case indicates that the robot has no knowledge of where it starts, we can model this by a probability distribution where each state has equal probability (a *uniform* distribution):

$$Prob_{t=0}(X = i) = 1/18.$$

Note the use of X as the random variable to indicate the position of the robot.

1.2.3 What is the belief after one move?

We denote the time after the move by time step 1, i.e. $t = 1$. Since the robot moved one tile to the right, the robot knows he could not be in the last tile initially. So, by case analysis on the motion we obtain the following distribution for the current state.

$$Prob_{t=1}(X = i) = \begin{cases} 1/17 & \text{if } i > 1 \text{ and } i \leq 18, \\ 0 & i = 1. \end{cases}$$

The case analysis also allows to conclude the robot cannot be at position $x = 1$ now.

1.2.4 What is the belief after one move and one observation?

The robot now observes a door. This makes a new point in time, i.e. $t = 2$. To update the belief when there is an observation we need to use Bayes Theorem. This is a way of factoring out the probability of being at position X given the observation O.

$$
\begin{aligned}
&Prob_{t=2}(X = x|O = \text{door}) = \\
&\quad Prob_{t=2}(O = \text{door}|X = x)Prob_{t=2}(X = x)/Prob_{t=2}(O = \text{door}).
\end{aligned}
$$

That is, the probability of $X = x$ given that we observe $O =$ door, is

1. the chances of seeing a door when it is at x

   i.e. $Prob_{t=1}(O = \text{door}|X = x)$, but this does not depend on the time since the map is fixed. Thus, $Prob_{t=1}(O = \text{door}|X = x) = Prob(O = \text{door}|X = x)$

2. and being at x

   (that is, $Prob_{t=2}(X = x)$, but $Prob_{t=2}(X = x) = Prob_{t=2}(X = x)$ since we do not move now)

3. modulated by the chances of seeing a door;

   that is, $Prob_{t=t_0}(O = \text{door})$, which are not constant and may change with time (for example, at the beginning, if the robot does not know anything, then there are 6 doors in the environment, so the $Prob_{t=0}(O = \text{door}) = 6/18 = 1/3$; however, after knowing one move to the right happened, we know we can not be in $x = 1$, so $Prob_{t=1}(O = \text{door}) = 6/17$.

However, the robot does not move from $t = 1$ to $t = 2$, so $Prob_{t=2}(O = \text{door}) = 6/17$.

In general, the belief at time $t = t_0 + 1$ can be found from the earlier time step (the earlier movement).

$$
\begin{aligned}
&Prob_{t=t_0+1}(X = x|O = \text{door}) = \\
&\quad Prob_{t=t_0}(O = \text{door}|X = x)Prob_{t=t_0}(X = x)/Prob_{t=t_0}(O = \text{door}).
\end{aligned}
$$

Thus, for example

Case $x = 1$**:** We know $Prob_{t=1}(X = 1) = 0$, thus

$$\begin{aligned}
& Prob_{t=2}(X = 1|O = \text{door}) \\
&= Prob(O = \text{door}|X = 1)Prob_{t=1}(X = 1)/Prob_{t=2}(O = \text{door}) \\
&= Prob(O = \text{door}|X = 1)0/Prob_{t=1}(O = \text{door}) \\
&= 0.
\end{aligned}$$

Case $x = 2$**:** We know, $Prob(O = \text{door}|X = 2) = 0$; that is, one cannot see a door in position 2 in this environment. Thus,

$$\begin{aligned}
& Prob_{t=2}(X = 2|O = \text{door}) \\
&= Prob(O = \text{door}|X = 2)Prob_{t=1}(X = 2)/Prob_{t=1}(O = \text{door}) \\
&= 0 \cdot (1/17)/Prob_{t=1}(O = \text{door}) \\
&= 0.
\end{aligned}$$

The fact that we cannot see a door from position $x = 1$ is part of the *sensor model*. That is, we model what are the chances of observing something from a certain position in the environment.

Clearly, if we are in a position x_i when there is no door, we know that $Prob(O = \text{door}|X = x_i) = 0$ and thus we can establish that $Prob_{t=2}(X = x_i) = 0$. Thus, all the positions where there is no door are analogous to the case $x = 1$. Namely,

$$Prob_{t=2}(X = x_i) = 0 \text{ for } x_i \in \{1, 2, 4, 6, 7, 9, 11, 12, 13, 15, 17, 18\}.$$

So, lets look at the case when there is a door.

Case $x = 3$**:** We know, $Prob(O = \text{door}|X = 3) = 1$; that is, once at $x = 3$ we certainly see a door.

$$\begin{aligned}
& Prob_{t=2}(X = 3|O = \text{door}) \\
&= Prob(O = \text{door}|X = 3)Prob_{t=1}(X = 3)/Prob_{t=1}(O = \text{door}) \\
&= 1 \cdot (1/17)/Prob_{t=1}(O = \text{door}) \\
&= \frac{1}{17 \cdot Prob_{t=1}(O = \text{door})} \\
&= \frac{17}{17 \cdot 6} = 1/6.
\end{aligned}$$

Thus, in summary we have the following.

$$Prob_{t=2}(X = x) = \begin{cases} 1/6 & x \text{ has a door },\\ 0 & x \text{ has no a door }. \end{cases}$$

1.2.5 What is the belief after two more moves?

Lets consider the situation of the robot after two moves the next time step $t = 3$. We do a case analysis again. Namely, $Prob_{t=3}(X = x)$ is described by analysing the exclusive ways to get here from a previous position. We see then that $Prob_{t=3}(X = x)Prob_{t=2}(X = x - 2)$ since the robot can get to position x only if the robot were at position $x - 2$ in the previous time step. This is a simple update then.

$$Prob_{t=3}(X = x) = \begin{cases} 1/6 & x - 2 \text{ has a door },\\ 0 & x - 2 \text{ has no a door }. \end{cases}$$

In terms of the actual values

$$Prob_{t=3}(X = x) = \begin{cases} 1/6 & x \in \{5, 7, 10, 12, 16, 18\}\\ 0 & \text{otherwise.} \end{cases}$$

1.2.6 What is the belief after two more moves and observing the door?

Now this becomes $t = 4$. We apply Bayes Theorem again, and the formula we derived before.

$$Prob_{t=4}(X = x|O = \text{door}) =$$
$$Prob_{t=3}(O = \text{door}|X = x)Prob_{t=3}(X = x)/Prob_{t=4}(O = \text{door}).$$

Now, there are lots of places where we do not observe a door and thus $Prob_{t=3}(O = \text{door}|X = x) = 0$. Also, there are many places we know we were not, and thus $Prob_{t=3}(X = x) = 0$ And also, two more moves right does no change $Prob_{t=4}(O = \text{door}) = Prob_{t=3}(O = \text{door})$. Thus, the probability is not zero only on those cases where we can see a door and we came from two position where there was a door. In summary, we now have the following.

$$Prob_{t=4}(X = x) = \begin{cases} 1/3 & x \in \{5, 10, 16\}\\ 0 & \text{otherwise.} \end{cases}$$

1.2.7 Repeat with a robot that can sense the color of the door

This is an adjustment to the sensor model. That is, what is changing is the term of the form

1. $Prob_{t=t_0}(O = \text{door})$ about the environment, and
2. $Prob_{t=t_0}(O = \text{door}|X = x)$ about the sensor model.

In particular,

1. $Prob_{t=t_0}(O = \text{door})$ would change to two types of expressions.
   (a) $Prob_{t=t_0}(O = \text{green door})$, that is how likely is to see a green door in the environment at t_0, which for example at $t = 0$ would be 3/18=1/6.
   (b) Similarly, $Prob_{t=t_0}(O = \text{red door})$ is the chances of seeing a red door (based on our knowledge of the map). This also is initially 1/6.
2. $Prob_{t=t_0}(O = \text{door}|X = x)$ changes also to two types of expressions.
   (a) $Prob_{t=t_0}(O = \text{green door}|X = x)$ changes of seeing a green door from position x, and this would be zero anywhere there is not a green door and 1 otherwise. This is,

$$Prob_{t=t_0}(O = \text{green door}|X = x) = \begin{cases} 1 & x \in \{5, 8, 16\} \\ 0 & \text{otherwise.} \end{cases}$$

   (b) Similarly, $Prob_{t=t_0}(O = \text{red door}|X = x)$ are the chances of seeing a red door from position x. This is 1 where there is a red door.

$$Prob_{t=t_0}(O = \text{red door}|X = x) = \begin{cases} 1 & x \in \{3, 10, 14\} \\ 0 & \text{otherwise.} \end{cases}$$

The new definitions of $Prob_{t=t_0}(O = \text{red door}|X = x)$ and $Prob_{t=t_0}(O = \text{green door}|X = x)$ are the new *sensor model.*

Now, we first need to revise the step $t = 2$, that is, when the robot has moved one step right and sees a door (that is red). At $t = 1$ all tiles expect position $x = 1$ are possible with probability 1/17. Applying the generic formulation of Bayes Theorem we have the following calculations.

Case $x = 3, x = 10, x = 14$**:** Because $Prob_{t=1}(O = \text{red door}|X = x) = 1 \neq 0$

$$
\begin{aligned}
&Prob_{t=2}(X = x|O = \text{red door}) = \\
&\quad Prob_{t=1}(O = \text{red door}|X = x)Prob_{t=2}(X = x)/Prob_{t=1}(O = \text{red door}) \\
&\quad 1 \cdot (1/17)/Prob_{t=2}(O = \text{red door}) \\
&\quad (1/17)/(3/17) = 1/3.
\end{aligned}
$$

Case $x \neq 3 \wedge x \neq 10 \wedge x \neq 14$**:** Because $Prob_{t=1}(O = \text{red door}|X = x) = 0$ we have

$$Prob_{t=2}(X = x|O = \text{red door}) = 0.$$

Then, the time $t = 3$ was easy because we just move (with certainty) two positions to the right. Therefore

$$Prob_{t=3}(X = x) = \begin{cases} 1/3 & x \in \{5, 12, 16\} \\ 0 & \text{otherwise.} \end{cases}$$

For the last observation, the robot observes a green door. So using the rule by Bayes Theorem, we get the following expression.

Case $x = 5, x = 16$**:** Because $Prob_{t=3}(O = \text{green door}|X = x) = 1 \neq 0$

$$
\begin{aligned}
&Prob_{t=4}(X = x|O = \text{green door}) = \\
&\quad Prob_{t=3}(O = \text{green door}|X = x)Prob_{t=3}(X = x)/Prob_{t=3}(O = \text{green door}) \\
&\quad 1 \cdot (1/3)/Prob_{t=3}(O = \text{green door})
\end{aligned}
$$

Now, it turns out that $Prob_{t=3}(O = \text{green door})$ is the probability of seeing a green door at time $t = 3$; which means we have seen a door earlier. Of the configurations of the map, there is 2 cases out of 3 where this could have happened. Thus the result is

$$\frac{(1/3)}{2/3} = \frac{1}{2}.$$

Case $x = 8:$ Because $Prob_{t=3}(O = \text{green door}|X = x) = 1 \neq 0$ but $Prob_{t=3}(X = x) = 0$ is zero.

Case $x \neq 5 \wedge x \neq 8 \wedge x \neq 16$ Because $Prob_{t=1}(O = \text{green door}|X = x) = 0$ we have

$$Prob_{t=4}(X = x|O = \text{green door}) = 0.$$

In summary

$$Prob_{t=4}(X = x) = \begin{cases} 1/2 & x \in \{5, 16\} \\ 0 & \text{otherwise.} \end{cases}$$

2 Seminar 2

2.1 Problem Description

There are two major differences on the localization problem from the earlier situation.

1. Initially, the robot knows that is is not at a tile where there is a door.
2. There is *actuator uncertainty*; namely, there is now no certainty about and action (or a move).

2.2 Solution Method

2.2.1 The initial belief

Since the robot knows that at time $t = 0$ there is no door, the robot knows (with equal probability) that it must be at a position without door. There are 6 of the 18 positions with a door. Or 12 positions with no door out of 18. Therefore,

$$Prob_{t=0}(X = x) = \begin{cases} 1/12 & x \in \{1, 2, 4, 6, 7, 9, 11, 12, 13, 15, 17, 18\} \\ 0 & \text{otherwise.} \end{cases}$$

2.2.2 The motion model

Now, a move has a probability of success. In fact, in the previous exercise we knew that the $Prob(\text{move forward}) = 1$ (except at position $x = 18$. We represent the outcome of a move as the following *motion model.*

Case move forward:

$$Prob(\text{move forward}|X = x) = \begin{cases} 2/3 & x \neq 18 \\ 0 & \text{otherwise} \end{cases}$$

Case move backward:

$$Prob(\text{move backward}|X = x) = \begin{cases} 1/6 & x \neq 0 \\ 0 & \text{otherwise} \end{cases}$$

Case stay:

$$Prob(\text{stay}|X = x) = 1/6.$$

2.2.3 What is the belief after one move?

This is time $t = 1$. The analysis of a move is by case analysis. We start with the position $t = 0$. The robot is in position $x = 1$ at time $t = 1$ if

1. the robot did not move **and** it was at $x = 1$ at time $t = 0$, **or (exclusively)**
2. the robot moved *back* **and** it was at $x = 2$ at time $t = 0$.

Then,

$$Prob_{t=1}(X = 1) = \begin{cases} & Prob_{t=0}(X = 1) \cdot Prob_{t=0}(\text{stay}) \\ + & Prob_{t=0}(X = 2) \cdot Prob_{t=0}(\text{move backward}). \end{cases}$$

In this case, this results in the following values.

$$Prob_{t=1}(X = 1) = \begin{cases} & 1/12 \cdot 1/6 \\ + & 1/12 \cdot 1/6. \end{cases}$$

Lets move now to the case position $x = 2$. The principle is simple.

1. the robot moved **and** it was at $x = 1$ at time $t = 0$, **or (exclusively)**
2. the robot did not move **and** it was at $x = 2$ at time $t = 0$, **or (exclusively)**
3. the robot moved *back* **and** it was at $x = 3$ at time $t = 0$.

This transform in the following description of the chances of being at position $x = 1$ at time $t = 1$.

$$Prob_{t=1}(X = 2) = \begin{cases} & Prob_{t=0}(X = 1) \cdot Prob_{t=0}(\text{move forwards}) \\ + & Prob_{t=0}(X = 2) \cdot Prob_{t=0}(\text{stay}) \\ + & Prob_{t=0}(X = 3) \cdot Prob_{t=0}(\text{move backward}). \end{cases}$$

In this case, we know the robot did not move backward from position 3 because we know $Prob_{t=0}(X = 3) = 0$. Replacing the numbers, we get the following.

$$Prob_{t=1}(X = 2) = \begin{cases} & 1/12 \cdot 2/3 \\ + & 1/12 \cdot 1/6 \\ + & 0 \cdot 1/6. \end{cases}$$

Lets do one more, the case position $x = 3$. In this case, the robot did not stay because it was not there.

$$\begin{aligned} Prob_{t=1}(X = 2) &= \begin{cases} & Prob_{t=0}(X = 2) \cdot Prob_{t=0}(\text{move forwards}) \\ + & Prob_{t=0}(X = 3) \cdot Prob_{t=0}(\text{stay}) \\ + & Prob_{t=0}(X = 4) \cdot Prob_{t=0}(\text{move backward}), \end{cases} \\ &= \begin{cases} & 1/12 \cdot Prob_{t=0}(\text{move forwards}) \\ + & 0 \cdot Prob_{t=0}(\text{stay}) \\ + & 1/12 \cdot Prob_{t=0}(\text{move backward}), \end{cases} \\ &= \begin{cases} & 1/12 \cdot 2/3 \\ + & 0 \cdot 1/6 \\ + & 1/12 \cdot 1/6. \end{cases} \end{aligned}$$

With these examples, we can now describe the general case.

$$Prob_{t=1}(X = x) = \begin{cases} & Prob_{t=0}(X = x - 1) \cdot Prob_{t=0}(\text{move forwards}) \\ + & Prob_{t=0}(X = x) \cdot Prob_{t=0}(\text{stay}) \\ + & Prob_{t=0}(X = x + 1) \cdot Prob_{t=0}(\text{move backward}). \end{cases}$$

In fact, the formula is quite general if we let $Prob_{t=0}(X = 0) = 0 = Prob_{t=0}(X = 19)$.

With this, the entire belief (distribution) $Prob_{t=1}(X = x)$ at time $t = 1$ is computed. Figure 2 shows the initial belief and the belief at $t = 1$. Note that there is already a peak in the distribution.

2.2.4 What is the belief after one move and we see a door (but cannot tell the color?

This is time $t = 2$ and what applies is Bayes Theorem again.

$$\begin{aligned} &Prob_{t=2}(X = x | O = \text{door}) = \\ &\quad Prob_{t=2}(O = \text{door} | X = x) Prob_{t=2}(X = x) / Prob_{t=2}(O = \text{door}). \end{aligned}$$

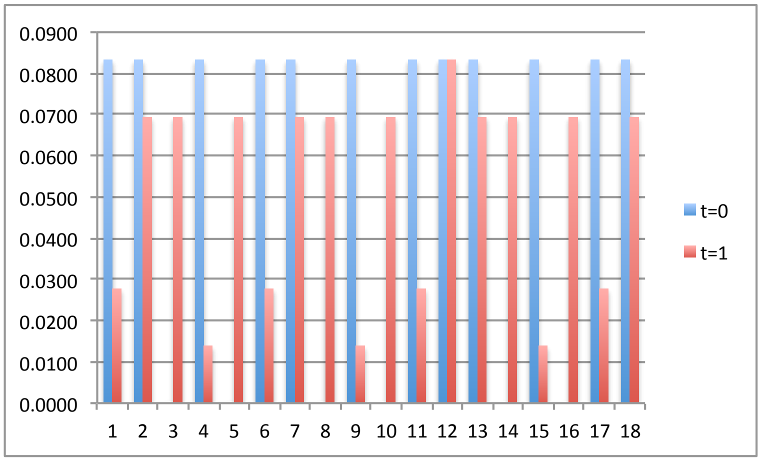

Figure 2: Contrast of the probability distributions at time $t = 0$ (the initial state) and time $t = 1$ when we have made one move.

Here, because we cannot tell the direction of motion, it is reasonable to assume then that $Prob_{t=2}(O = \text{door})$ remains constant. The term $Prob_{t=2}(O = \text{door}|X = x)$ is $Prob_{t=1}(O = \text{door}|X = x)$ as there is no move between $t = 1$ and $t = 2$. Similarly $Prob_{t=2}(X = x) = Prob_{t=1}(X = x)$. Thus, the formula can now be used to estimate the distribution at $t = 2$ from the distribution at $t = 1$.

$$Prob_{t=2}(X = x|O = \text{door}) = \\ Prob_{t=1}(O = \text{door}|X = x)Prob_{t=1}(X = x)/Prob_{t=1}(O = \text{door}).$$

Moreover, in this case we know there are only 6 positions where $Prob_{t=1}(O = \text{door}|X = x) = 1$ and all the others are zero. These positions is where there is a door and thus, only six positions will have $Prob_{t=2}(X = x|O = \text{door}) \neq 0$. It turns out that, in the positions where there is a door, also $Prob_{t=1}(X = x)$ takes the same value. Thus, we have the following summary.

$$Prob_{t=2}(X = x) = \begin{cases} 1/6 & x \text{ has a door }, \\ 0 & x \text{ has no a door }. \end{cases}$$

2.2.5 What is the belief after two more moves in the blind

We let $t = 3$ be the time after the next move. We will compute the belief now (and then $t = 4$ will be the subsequent move). The formula for figuring

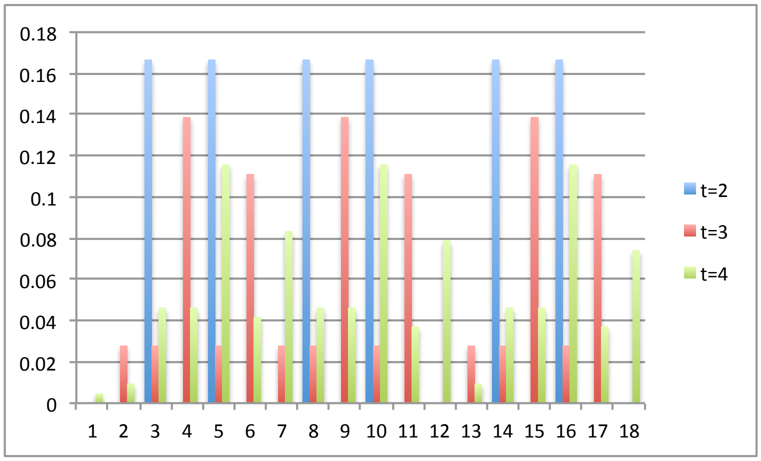

Figure 3: Contrast of the probability distributions at time $t = 2$, $t = 3$ and $t = 4$ as the robot moves without sensor feedback.

out the move is simply the case analysis we did before, but adapted to $t = 3$.

$$Prob_{t=3}(X = x) \quad = \quad \left\{ \begin{array}{ll} & Prob_{t=2}(X = x-1) \cdot Prob_{t=0}(\text{move forwards}) \\ + & Prob_{t=2}(X = x) \cdot Prob_{t=0}(\text{stay}) \\ + & Prob_{t=2}(X = x+1) \cdot Prob_{t=0}(\text{move backward}). \end{array} \right.$$

The second move applies this recurrence one more time, adjusted for $t = 4$.

$$Prob_{t=4}(X = x) \quad = \quad \left\{ \begin{array}{ll} & Prob_{t=3}(X = x-1) \cdot Prob_{t=0}(\text{move forwards}) \\ + & Prob_{t=3}(X = x) \cdot Prob_{t=0}(\text{stay}) \\ + & Prob_{t=3}(X = x+1) \cdot Prob_{t=0}(\text{move backward}). \end{array} \right.$$

The following plot (see Figure 3) shows the distribution after this two extra steps. We note that the high values of the six position at time $t = 2$ progressively loose strength as the robot moves *in the dark*; that is, since feedback from the sensors. In this sense the uncertainty of the motion propagates to uncertainty about the current position. The belief losses intensity, but it also shifts to the right, as expected.

Table 1: The values after one move, seeing one door, and another move ($t = 3$).

x	$P_{t=3}(X = x)$
1	0
2	0.0278
3	0.0278
4	0.1388
5	0.0278
6	0.1111
7	0.0278
8	0.0278
9	0.1388
10	0.0278
11	0.1111
12	0
13	0.0278
14	0.0278
15	0.1388
16	0.0278
17	0.1111
18	0

2.2.6 What is the belief after two more moves but after the first there is no door and after the second the robot observes a door

In this case we take over from $t = 3$ earlier and we apply a time $t = 4$ sensor update, which consists of applying Bayes Theorem. We adjust it for observing no door. The case $t = 3$ has the values in Table 1

$$Prob_{t=4}(X = x|O = \text{no door}) = \\ Prob_{t=3}(O = \text{no door}|X = x)Prob_{t=3}(X = x)/Prob_{t=3}(O = \text{no door}).$$

There are six positions where $Prob_{t=3}(O = \text{no door}|X = x) = 0$. A simple way to do this is to simply consider the equivalent recurrence

$$Prob_{t=4}(X = x|O = \text{no door}) = \\ \rho \cdot Prob_{t=3}(O = \text{no door}|X = x)Prob_{t=3}(X = x)$$

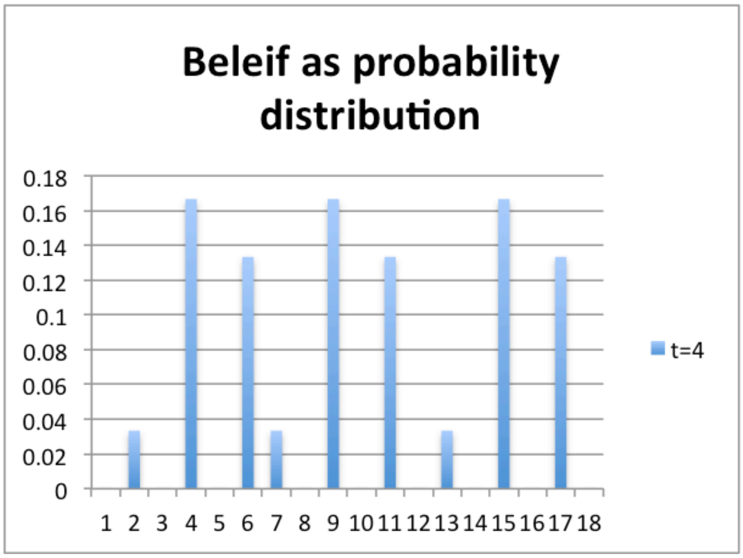

Figure 4: The belief at $t = 4$, when we have moved once, seen a door once, moved again, and then seen no door.

with ρ a normalization factor. But then we compute

$$N[Prob_{t=4}(X = x|O = \text{no door})] = \\ Prob_{t=3}(O = \text{no door}|X = x)Prob_{t=3}(X = x)$$

And later we normalize

$$Prob_{t=4}(X = x|O = \text{no door}) = \\ \frac{N[Prob_{t=4}(X = x|O = \text{no door})]}{\sum_{y \in X} N[Prob_{t=4}(X = y|O = \text{no door})]}$$

The result is shown in Figure 4. Certainly, the robot is not in front of a door. However, it is already more likely to be just one cell after a door that is the first (in a pair fo doors separated by one tile). This is because we may have started in the middle cell of those two doors separated by one cell, and there is chances of going back and forward. Also there is a small chance we are one cell before a door.

Now, at $t = 5$ we move forwards, and at $t = 6$ we observe a door. Thus, for $t = 5$ we apply the case analysis, and for $t = 6$ we will apply Bayes Theorem.

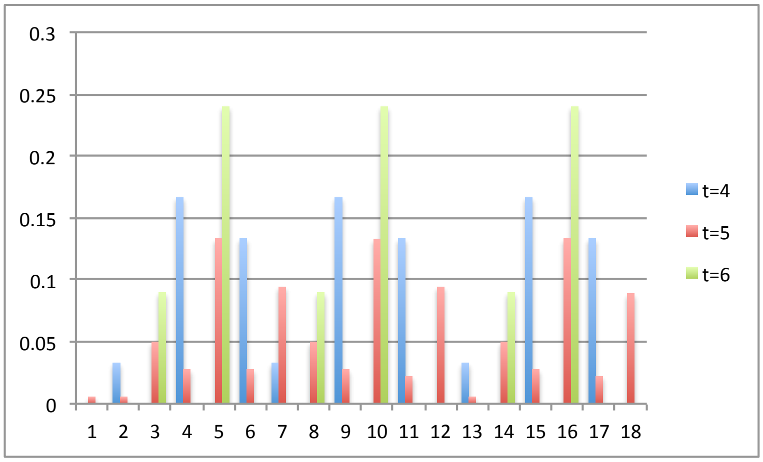

Figure 5: Evolution of the belief after $t = 4$, when the robot moves ahead ($t = 5$) and then sees a second door $t = 6$.

The case analysis for $t = 5$ uses the motion model (the probabilities of the moves) and follows.

$$Prob_{t=5}(X = x) \quad = \quad \begin{cases} & Prob_{t=4}(X = x - 1) \cdot Prob_{t=0}(\text{move forwards}) \\ + & Prob_{t=4}(X = x) \cdot Prob_{t=0}(\text{stay}) \\ + & Prob_{t=4}(X = x + 1) \cdot Prob_{t=0}(\text{move backward}). \end{cases}$$

While for the Bayes Theorem, we apply the formula

$$N[Prob_{t=6}(X = x | O = \text{door})] = \\ Prob_{t=5}(O = \text{door} | X = x) Prob_{t=5}(X = x)$$

because we now see a door (and normalize — which just makes sure the probabilities add up to one). Figure 5 shows the evolution of the belief in this 3 time steps. We can see that, at time $t = 6$, the probability of being in front of the second door of a group (door, empty, door) is about 24%. But the probability of being at the first door is about 9%. So, the robot keeps a reservation for being back at the first of the observed doors.

2.2.7 What if the robot can distinguish the color?

In this case, the first door observed at time $t = 2$ is red, while the door observed at $t = 6$ is green. However, we have the same situation where now we have a more specialized *sensor model*.

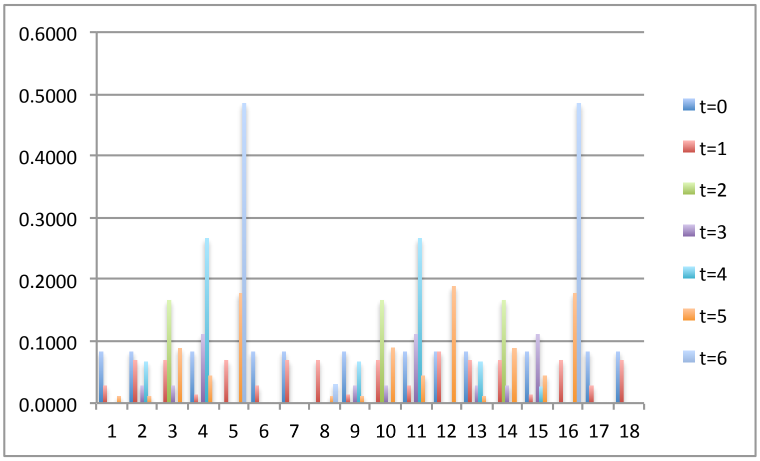

Figure 6: Evolution with a color sensor of the belief from $t = 0$, until the second door $t = 6$ (green), while at $t = 2$ a red door is observed.

This is analogous to the change in Seminar 1 from a color-blind sensor to a sensor that can detect colors. See section 1.2.7 for the definitions of the color sensor model. For now suffice to say that the same method is used with these sensor models applied now at the time $t = 2$ when a red door is observed and at $t = 6$ when a green door is observed. Figure 6 is the evolution from the initial belief $t = 0$ when all empty tiles are believed with equal probability until $t = 6$ the second door (green) is observed. At $t = 2$ the robot knows it is in front of one of the three red doors with equal probability despite the uncertainty of the moves. At $t = 6$, the robot believes it is at locations 5 and 16 with 48.5% chances. Not completely sure, because there is a small chance to be in front of the green door at location 8, by starting at 11 and having all moves a backward move.

3 Seminar 3

3.1 Problem statement

Now we also have uncertainty on what the sensors obtain. This is normally the case, sensors have error or noise. The exercise asks to handle the situation when if there is a red-door, the sensors report it as red 75% of the time, but green 25% of the time. Similarly, a green-door appears to green 75% of the time, but red 25% of the time.

3.2 Solution Method

In fact the solution methods is not very different than before, however now we have a different sensor model than in Section 1.2.7. In front of a green door, we see it as green only 75% of the time revises the sensor model of Section 1.2.7.

$$Prob_{t=t_0}(O = \text{green door}|X = x) = \begin{cases} 3/4 & x \in \{5, 8, 16\} \\ 1/4 & x \in \{3, 10, 14\} \\ 0 & \text{otherwise.} \end{cases}$$

This is because in $\{5, 8, 16\}$ (there is a green door), we will only see it green with probability 3/4. However, in front of a red door ({ 3, 10, 14 }) we see it as green 1/4 of the time. We never see a phantom green door if there is no door at all.

Similarly, the sensor model must also be adapted for when a red door is visible.

$$Prob_{t=t_0}(O = \text{red door}|X = x) = \begin{cases} 3/4 & x \in \{3, 10, 14\} \\ 1/4 & x \in \{5, 8, 16\} \\ 0 & \text{otherwise.} \end{cases}$$

The rationale for this sensor model is analogous. Once this sensor model is established, the method applies as well. A motion step is incorporated by the case analysis.

$$Prob_{t=t_0+1}(X = x) \quad = \quad \begin{cases} & Prob_{t=t_0}(X = x - 1) \cdot Prob(\text{move forwards}) \\ + & Prob_{t=t_0}(X = x) \cdot Prob(\text{stay}) \\ + & Prob_{t=t_0}(X = x + 1) \cdot Prob(\text{move backward}) \end{cases}$$

where now we have indicated the motion model does not depend on the time.

Once we have established the belief after the motion, we apply Bayes Theorem for updating the belief for a sensor reading. Suppose the sensor reading is s.

$$N[Prob_{t=t_0+1}(X = x|O = s)] = \\ Prob_{t=t_0}(O = s|X = x) Prob_{t=t_0}(X = x)$$

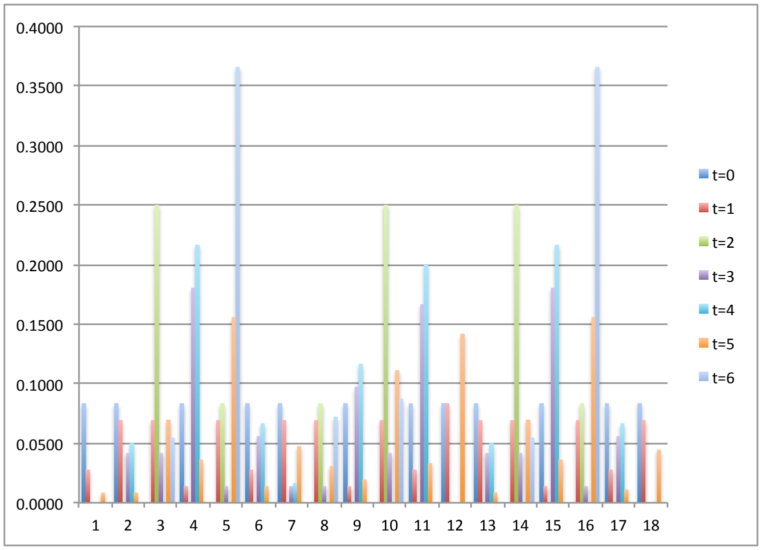

Figure 7: Evolution with a color sensor of the belief from $t = 0$, until the second door $t = 6$ (green), while at $t = 2$ a red door is observed.

and then normalize. The normalization is a simplification of the term $Prob_{t=t_0}(O = s)$ which in fact is $Prob_{t=t_0}(O = s|$ history) which in fact is the probability of observation $O = s$ given the map and the history of moves and observations until time t_0. But is reasonable to assume the most recent observations is far more relevant than any earlier ones. The result of the evolution is now shown in Figure 7. Now the two positions in front of a green door at 5 and 16 have about 36.6% of being the current position. In fact, the noise color sensor makes possible to be in front of any of the positions where there was a door. The positions $x = 3$ and $x = 14$ despite holding a red door have 5% chances of being the current position despite the sensor just reported a green door. And of course, the positions of the green door $x = 10$ has in itself about 8%. The position $x = 8$ also in itself has about 7